Geel 2000 Language Schools

Math Department

Second Term

Sec. 2

2024/2025

Name :-----------------------------------

Class:-----------------------------------

Unit 1 : Sequences and series

Unit 1 : Lesson 1 : Sequences

The sequence is a function whose domain is the set of the positive integers Z+ or a subset of it and its range is a set of the real numbers R where the first term is denoted by T1, the second term is denoted by T2, and the third term is denoted by T3 and son on.... and the nth term is denoted by Tn the sequence can be expressed by writing down its terms between two brackets as follows: (T1 , T2 , T3 , ... , Tn) or denoted by the symbol (Tn).

Finite and Infinite Sequences

The sequence is finite if the number of its terms is finite (i.e. can be counted) the sequence is infinite if the number of its terms is infinite (an infinite number of elements)

Ex 1 :

Write down each of the sequences whose n^{th} term is given by the relation:

a $T_n = n^2 - 1$ (to five terms starting with first term)

b $T_n = \frac{(-1)^n}{2n+1}$ (to an infinite number of terms starting with first term)

General Term of a Sequence

For example:

- The general term of the sequence of even numbers: (2 , 4 , 6 , 8 , ...) is $T_n = 2n$
- The general term of the sequence of odd numbers:(1 , 3 , 5 , 7 , ...) is $T_n = 2n - 1$
- The general term of the sequence:($\frac{-1}{3}$, $\frac{1}{4}$, $\frac{-1}{5}$, $\frac{1}{6}$, ...) is $T_n = \frac{(-1)^n}{n+2}$

Increasing and Decreasing Sequences

Check the following sequences:

1) $(-5\ ,\ -1\ ,\ 3\ ,\ 7\ ,\ 11\ ,\ 15\ ,\ ....)$ **(What do you notice?)**

2) $(4\ ,\ 2\ ,\ 1\ ,\ \frac{1}{2}\ ,\ \frac{1}{4}\ ,\ \frac{1}{8}\ ,\ ....)$ **(What do you notice?)**

- **In the first sequence:** $-1 > -5$ i.e. $T_2 > T_1$, $3 > -1$ i.e. $T_3 > T_2$ and so on for the rest of the terms. I.e. each term of the sequence is greater than the directly previous term.

- **In the second sequence:** $2 < 4$ i.e. $T_2 < T_1$, $1 < 2$ i.e. $T_3 < T_2$ and so on i.e. each term of the sequence is lesser than the directly previous term.

Definition:

- The sequence (T_n) is called **increasing** if $T_{n+1} > T_n$
- The sequence (T_n) is called **decreasing** if $T_{n+1} < T_n$

Ex 2 :

Show which of the sequences (T_n) is in increasing , decreasing or otherwise.

a $T_n = 2n + 3$ **b** $T_n = \frac{1}{3n - 1}$ **c** $T_n = \frac{(-1)^n}{2n} + 4$

Ex 3 :

Write down the general term for each of the following sequences:

a $(2\ ,\ 5\ ,\ 8\ ,\ 11\ ,\ ......)$ **b** $(\frac{1}{2}\ ,\ \frac{2}{3}\ ,\ \frac{3}{4}\ ,\ \frac{4}{5}\ ,\ ......)$

Lesson 2 : Series and summation notation

Finite Series

It is written in the form: $T_1 + T_2 + T_3 + ... + T_r + ... + T_n$ where n is a positive integer, T_n is the terms whose order is n in the series and the numerical value of the finite series is called the sum of the terms of the corresponding sequence.

The finite series: $a_1 + a_2 + a_3 + ... + a_r \; ... + a_n$ can be written in the form $\sum_{r=1}^{n} (a_r)$ and read as the sum of a_r from $r = 1$ to $r = n$

Ex 1 :

Expand each of the following series, then find the expansion sum.

a $\sum_{r=1}^{4} (r^2)$ **b** $\sum_{r=1}^{7} (2r - 1)$ **c** $\sum_{r=1}^{n} \left(\frac{1}{r+1} - \frac{1}{r}\right)$

Infinite Series

The terms of the infinite series cannot be counted . For example the series:
$-3 + 9 - 27 + 81 - 243 + \dots$ can be written in the form of $\sum_{r=1}^{\infty} (-3)^r$. The symbol ∞ is used to express infinity.

Ex 2 :

Use the summation notation Σ to write down the series: $2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$

Algebraic Properties of Summation

1- If (T_r) and (E_r) are two sequences, $n \in Z^+$ and $C \in R$, then:

a $\sum_{r=1}^{n} C = C\,n$

b $\sum_{r=1}^{n} cT_r = c \sum_{r=1}^{n} T_r$

c $\sum_{r=1}^{n} (T_r \pm E_r) = \sum_{r=1}^{n} T_r \pm \sum_{r=1}^{n} E_r$

2- $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$, $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

Ex 3 :

Find in two different methods $\sum_{r=1}^{4} (3 - 2r + r^2)$

Lesson 3 : Arithmetic sequences

Definition 1

Arithematic Sequence

The arithmetic sequence is the sequence in which the difference between a term and the directly previous term to it equals a constant and it is called the common difference of the sequence. it is denoted by the symbol (d)

i.e.: $d = T_{n+1} - T_n$ for each $n \in Z^+$

Ex 1 :

Which of the following is an arithmetic sequence? why ?

a (7 , 10 , 13 , 16 , 19)

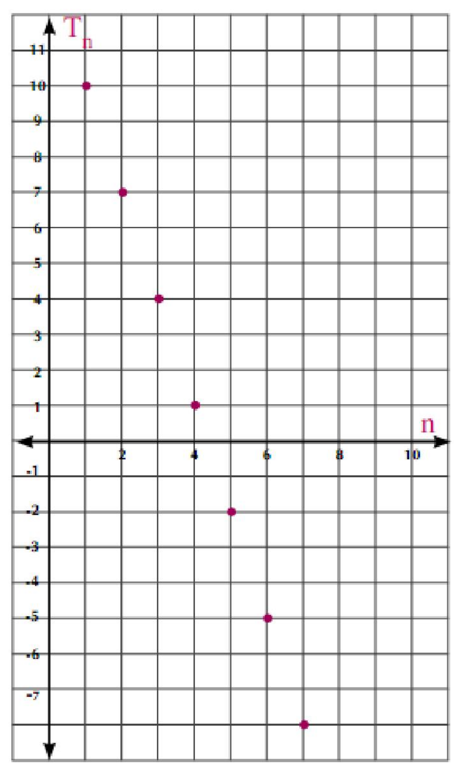
b (27 , 23 , 19 , 15 , 11, ...)

c $(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6})$

Graphical representation of an arithematic sequence

Ex 2 :

Find the next four terms of the arithmetic sequence (10, 7, 4, ...), then represent the seven terms graphically.



The n^{th} Term of the arithmetic sequence

From difinition (1) the n^{th} term of the arithmetic sequence (T_n) whose first term is a and common difference is d can be deduced as follows :

$T_1 = a$, $T_2 = a + d$ and $T_3 = a + 2d$ and by keeping this pattern, we find that the n^{th} term of this sequence is :

$\boxed{T_n = a + (n-1)d}$ and if $T_n = \ell$ (where ℓ is the last term, then $\boxed{\ell = a + (n-1)d}$

Ex 3 :

In the arithmetic sequence (13 , 16 , 19 , ... , 100)

a Find the tenth term. **b** Find the number of the terms of the sequence.

Ex 4 :

Find the number of the terms of the arithmetic sequence (7 , 9 , 11 , ... , 65) then find the value of the tenth term from the end.

Identifying the Arithmatic Sequence

The arithmetic sequence can be identified when its first and common difference are known.

Ex 5 :

If the seventh and fifteenth terms of an arithmetic sequence are 18 and 34 respectively, find the common difference and the first term then find the n^{th} term of this sequence.

Ex 6 :

Find the arithmetic sequence whose sixth term $= 17$ and the sum of its third and tenth terms $= 37$.

Arithmtic means

When there are two non-consecutive terms in an arithmetic sequence, then all the terms lying between those two terms are called arithmetic means. This concept can be used to find the missing terms between those two terms in the arithmetic sequence.

Definition 2

If a , b and c are three consecutive terms in an arithmetic sequence, then b is called the arithmetic mean between the two terms a and c where b- a = c-b,

i.e.: $2b = a + c$ then $b = \frac{a+c}{2}$ **So:** $\left(a , \frac{a+c}{2}, c\right)$ **is an arithmetic sequence.**

several arithmetic means: x_1 , x_2 , x_3 , ... , x_n can be inserted between the two numbers a and b in a way that the numbers : $(a , x_1 , x_2 , x_3 , ... , x_n, b)$ **form an arithmetic sequence.**

Inserting a Finite Number of Arithmatic Means Between Two Numbers

Ex 7 :

Insert 5 arithmetic means between 6 and 48

Ex 8 :

Insert seven arithmetic means between the two numbers – 24 and 16

Ex 9 :

Find the order and value of the first negative term in the arithmetic sequence (67, 64 , 61 , ...)

Ex 10 :

Find the order and value of the first term whose value is greater than 180 in the arithmetic sequence:

Lesson 4 : Arithmetic series

The Sum of the First n Terms of an Arithmetic Sequence

First: the sum of n^{th} terms of an arithmetic sequence in terms of its first term(a) and the last term (ℓ)

its common difference d, the number of its terms is denoted by the symbol S_n and is given by the following series:

$$S_n = a + (a + d) + (a + 2d) + \dots + (\ell - d) + \ell \qquad (1)$$

The series can be also written in the form:

$$S_n = \ell + (\ell - d) + (\ell - 2d) + \dots + (a + d) + a \qquad (2)$$

By adding the two equations (1) and (2), we deduce that:

$2S_n = (a + \ell) + (a + \ell) + (a + \ell) + \dots + (a + \ell)$ to n times

i.e. $2S = n(a + \ell)$ by dividing the two sides by 2 $\boxed{S_n = \frac{n}{2}(a + \ell)}$

Ex 1 :

Find $\sum\limits_{r=5}^{24} (4r - 3)$

Ex 2 :

Find:

a $\sum_{K=1}^{20} (6K + 5)$

b $\sum_{m=7}^{32} (12 - 5m)$

Second: Finding the sum of n^{th} terms of an arithmetic sequence in terms of its first term and its common difference.

You know that $\ell = a + (n - 1)\, d$ and $S_n = \frac{n}{2}(a + \ell)$

by substituting from the first relation in the second relation, then:

$S_n = \frac{n}{2}[a + a + (n - 1)\, d]$ **i.e.:** $\boxed{S_n = \frac{n}{2}[2a + (n - 1)\, d]}$

Ex 3 :

In the arithmetic series 5 + 8 + 11 + ... find:

- **a** The sum of its first twenty terms of the series .
- **b** The sum of ten terms starting from the seventh term .
- **c** The sum of the sequence terms starting from T_{10} up to T_{20}

Ex 4 :

In the arithmetic sequence (9 , 12 , 15 , ...), find :

a) The sum of its first fifteen terms .

b) The sum of the sequence terms starting from the fifth term up to the fifteenth term.

c) The number of terms whose sum equals 750 starting from the first term .

Ex 5 :

Find the arithmetic sequence in which:

(a) $T_1 = 23$, $T_n = 86$, $S_n = 545$

(b) $T_1 = 17$, $T_n = -95$, $S_n = -585$

Ex 6 :

In the arithmetic sequence (25 , 23 , 21 , ...), find:

a) The greatest sum of the sequence.

b) The number of terms whose sum = 120 starting from the first term " Explain the existance of two answers".

Lesson 5 : Geometric sequences

Definition 1

The sequence (T_n) where $T_n \neq 0$ is called a geometric sequence if $\frac{T_{n+1}}{T_n}$ = a constant for each $n \in Z^+$

The constant is called the common ratio of the sequence and is denoted by the symbol (r)

Ex 1 :

Show which of the following sequences (T_n) is geometric , then find the common ratio of each :

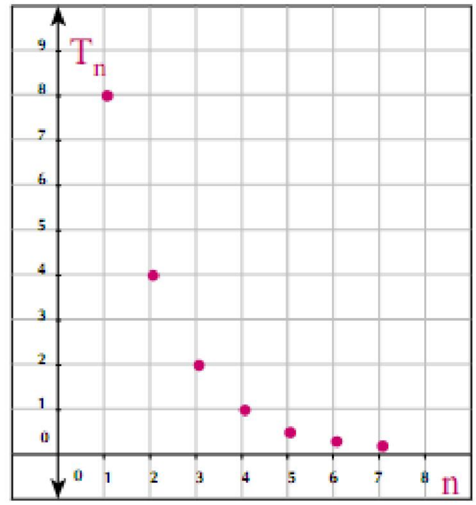
a $(T_n) = (2 \times 3^n)$ **b** $(T_n) = (4\, n^2)$

c The sequence (T_n) where: $T_1 = 12$,$T_n = \frac{1}{4} \times T_{n-1}$ (where $n > 1$)

Graphical Representation of The Geometric Sequence

Ex 2:

Find the next four terms of the geometric sequence (8, 4, 2, ...) then represent the first seven terms graphically.



The n th Term of the geometric sequence

From definition (1) the n^{th} term of the geometric sequence (T_n) whose first term is a and common ratio is r can be deduced as follows:

$T_1 = a$, $T_2 = ar$ and $T_3 = ar^2$, by continuing this pattern, we find that the n^{th} term of this sequence is: $\boxed{T_n = a\, r^{n-1}}$

Ex 3:

In the geometric sequence (2 , 4 , 8 , ...), find:

a The fifth term

b the order of the term whose value is 512

Identifying The Geometric Sequence

The geometric sequence can be identified whenever its first and common ratio are known (given).

Ex 4 :

(T_n) is a geometric sequence and all of its terms are positive. If $T_3 + T_4 = 6T_2$, $T_7 = 320$, find this sequence.

Geometric Means

Definition 2

If a, b and c are three successive terms of a geometric sequence, then b is known as the geometric mean between the two numbers a and c where: $\frac{b}{a} = \frac{c}{b}$

i.e. $b^2 = a\,c$ then $b = \pm\sqrt{a\,c}$

Tip

the geometric mean of a set of positive real values $T_1, T_2, T_3, \ldots, T_n$ is known as the n^{th} root of the product of these values. I.e. the geometric mean = $\sqrt[n]{T_1 \times T_2 \times \ldots \times T_n}$

Inserting a number of geometric means between two known quantities:

Ex 5 :

Find the geometric means of the sequence: (4 , ... , ... , ... , ... , ... , 2916)

Ex 6 :

Insert six geometric means between $\frac{1}{4}$ and 32

The relation between the arithmetic and geometric means of two numbers:

the arithmetic mean of two different positive real numbers is greater than their geometric mean.

Ex 7 :

If $6a$, $3b$, $2c$, $2d$ are positive quantities in an arithmetic sequence, prove that $bc > 2ad$

Lesson 6 : Geometric series

Sum of the First n Terms of a Geometric Series

First: To find the sum of n terms of a geometric series in terms of its first term and common ratio:

If $a + ar + ar^2 + \ldots + ar^{n-1}$ is a geometric series whose fist term is a and its common ratio is r , then the sum of S_n of this series can be found as follows:

$$S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1} \qquad (1)$$

By multiplying the two sides by r then:

$$rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n \qquad (2)$$

By subtracting the two equations, then:

$$S_n - rS_n = a - ar^n$$

i.e.: $S_n(1 - r) = a(1 - r^n)$

By dividing the two sides by $(1 - r)$ in a condition $1 - r \neq 0$

$$\boxed{S_n = \frac{a(1 - r^n)}{1 - r}, \; r \neq 1}$$

Ex 1 :

Find the sum of the geometric sequence in which : $a = 3$, $r = 2$, $n = 8$

Ex 2 :

Find the sum of the following two geometric sequences in which:

a $a = 4$, $r = 3$, $n = 6$

b $a = 1000$, $r = \frac{1}{2}$, $n = 10$

Second: to find the n terms of a geometric series in terms of its first and last terms

We know that: $S_n = \dfrac{a - a r^n}{1 - r}$ **(1)**

and: $\ell = a r^{n-1}$ by multiplying the two sides by r

then $\ell r = a r^n$ **(2)**

by substituting from **(2)** in **(1)** then: $\boxed{S_n = \dfrac{a - \ell r}{1 - r}, r \neq 1}$

Ex 3 :

Find the sum of the geometric series : $1 + 3 + 9 + ... + 6561$

Ex 4 :

Find the sum of the following two geometric sequences:

a $a = 9$, $r = 3$, $\ell = 6561$

b $a = 2048$, $r = \frac{1}{2}$, $\ell = 128$

Using the Summation Notation

Ex 5:

Find $\sum_{r=5}^{12} 3(2)^{r-1}$

Forming the geometric sequence

Ex 6 :

If the sum of the first n terms of a geometric sequence is given by the rule: $S_n = 128 - 2^{7-n}$, find the sequence and its seventh term.

Infinite geometric series

Definition 2

The infinite geometric series has an infinite number of terms. If their sum is a real number, the series is convergent because its sum gets near to a real number . If the series doesnot have a sum , it is divergent

Ex 7 :

Which of the following series can you sum an infinite number of its terms ? Explain

a) $75 + 45 + 27 + ...$

b) $24 + 36 + 54 + ...$

Sum of Infinite Geometric Series

We knew that the sum of n terms of the geometric series is given by the relation $S_n = \frac{a(1 - r^n)}{1 - r}$

and when we sum an infinite number of its terms, then r^n gets near to zero when $-1 < r < 1$

and the sum is: $\boxed{S_\infty = \frac{a}{1 - r}}$

Ex 8:

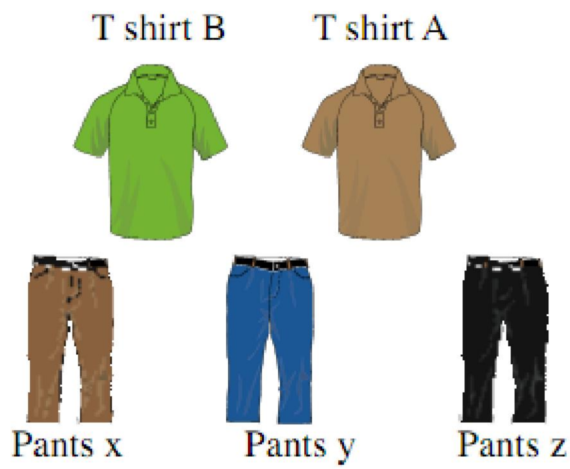
Find the sum for each of the following two geometric series if found:

a) $\frac{81}{8} + \frac{27}{4} + \frac{9}{2} + ...$

b) $\frac{2}{3} + \frac{5}{6} + \frac{25}{24} + ...$

Unit 2 : Permutations, Combinations

Lesson 1 : Counting Principle



Ex 1 :

The number of ways of sitting 4 students on 4 seats in a row equals :

Ex 2 :

How many three -digit numbers can be formed from the elements {2, 3, 5}?

Ex 3 :

How many different four - digit numbers can be formed from the elements {2, 3, 6, 8} so that the unit digit is 6?

Lesson 2 : Permutations

Definition 1

Factorial: The factorial of a positive integer n is written as $\underline{\lfloor n}$ and equals the product of all the positive integers which are lesser than or equal n where:

$$\underline{\lfloor n} = n\,(n-1)\,(n-2)\ldots 3 \times 2 \times 1$$

- When $n = 0$ **then** $\underline{\lfloor 0} = 1$
- When $n = 1$ **then** $\underline{\lfloor 1} = 1$
- $\underline{\lfloor 4} = 4 \times 3 \times 2 \times 1 = 4\,\underline{\lfloor 3}$,

  $\underline{\lfloor 6} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6\,\underline{\lfloor 5}$

In general: $\underline{\lfloor n} = n\,\underline{\lfloor n-1}$ where $n \in Z^{+}$

Ex 1 :

a) Find $\dfrac{\underline{\lfloor 10}}{\underline{\lfloor 8}}$

b) If $\underline{\lfloor n} = 120$ find the value of n

Ex 2 :

Find: a) $\dfrac{\underline{\lfloor 15}}{\underline{\lfloor 12}}$ b) $\dfrac{\underline{\lfloor 7}}{\underline{\lfloor 5}} + \dfrac{\underline{\lfloor 9}}{\underline{\lfloor 7}}$

Ex 3 :

Find the solution set of the equation:- $\frac{\lfloor n}{\lfloor n-2} = 30$

Permutations

Definition 1

The number of permutations of n different objects taking r at a time is denoted by the symbol $^{n}p_{r}$ where:

$^{n}p_{r} = n(n-1)(n-2)\ldots(n-r+1)$ where $r \leqslant n$, $r \in N$, $n \in Z^{+}$

$^{n}p_{0} = 1$

Ex 4 :

Find the value for reach of the following:

a $^{7}p_{4}$ b $^{4}p_{4}$ c $^{4}p_{3}$

Ex 5 :

Calculate the value of the following:

a $^{5}p_{2} + {}^{6}p_{3}$ b $\frac{^{5}p_{5}}{^{5}p_{4}}$

Ex 6 :

Find the number of the different ways, for 5 students to sit on 7 seats in one row.

Ex 7 :

How many ways can 4 persons sit on 4 seats in the form of a circle ?

Ex 8 :

If ${}^{7}p_{r} = 840$, find the value of $\lfloor\underline{r - 4}$

Ex 9 :

Find the value of the following:

a) $\underline{\lfloor 7} \div \underline{\lfloor 5}$

b) $3\,\underline{\lfloor 2} - \underline{\lfloor 3}$

c) ${}^{5}p_{3} \times \underline{\lfloor 2}$

d) ${}^{3}p_{3} \times {}^{2}p_{2}$

e) ${}^{8}p_{1} + {}^{8}p_{2}$

f) ${}^{7}p_{0} + {}^{7}p_{7}$

Lesson 3 : Combinations

Combinations

Definition

The number of combinations formed from r of objects chosen from n elements at the same time is ^nC_r where, $r \leqslant n$, $r \in N$, $n \in Z+$

$$^nC_r = \frac{^np_r}{\lfloor r}$$

Ex 1 :

Find the value of each of the following

a $^7C_5$

b $^7C_2$ (what do you notice)?

$$^nc_r = \frac{\lfloor n}{\lfloor r \, \lfloor n - r}$$

$$^nC_r = {^nC}_{n-r}$$

Ex 2 :

If $^{28}C_r = {^{28}C}_{2r-5}$, then find the value of r.

Ex 3 :

7 people have participated in a chess game so that a game is held between each two players. How many matches are there?

Ex 4 :

How many ways can a committee of two men and a woman be selected out of 7 men and 5 women?

Ex 5 :

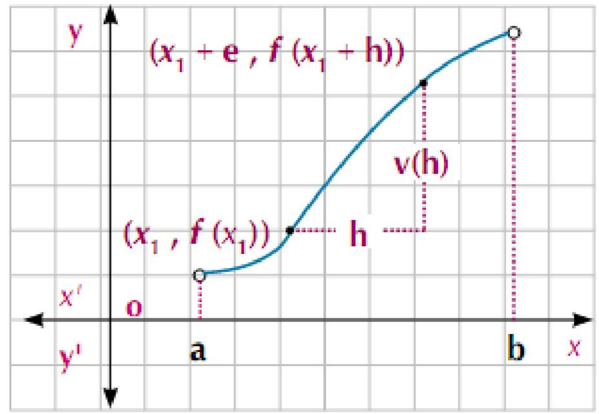
If ${}^{n}C_{3} = 120$, find the value of ${}^{n}C_{n-9}$

Unit 3 : Calculus

Lesson 1 : Rate of change

Function of Variation

$V(h) = f(x_1 + h) - f(x_1)$



Ex 1 :

If $f(x) = 3x^2 + x - 2$

and x varies from 2 to 2 + h , find the function of variation V, then calculate the change in f when:

a h = 0,3 **b** h = - 0,1

Ex 2 :

If $f(x) = x^2 - x + 1$, find the function of variation V when $x = 3$, then calculate:

a V(0.2)

b V(- 0.3)

Average Rate of Change Function

By dividing the function of variation v by h where $h \neq 0$, we get a new function a called the average rate of change function in f when $x = x_1$ where :

$$A(h) = \frac{v(h)}{h} = \frac{f(x_1 + h) - f(x_1)}{h} \quad \text{or} \quad \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Ex 3 :

If $f : [0, \infty[\longrightarrow \mathbb{R}$ where $f(x) = x^2 + 1$, find :

a The average rate of change function in f when $x = 2$, then calculate A (0,3)

b The average rate of change in f when x varies from 3 to 4

Ex 4 :

If $f(x) = x^2 + 3x - 1$, find:

a The average rate of change function when $x = 2$, then find a (0.2)

b The average rate of change when x varies from 4.5 to 3

Rate of Change Function

If $f:]a, b[\longrightarrow \mathbb{R}$ where $y = f(x)$ and $x_1, x_1 + h \in]a, b[$, then :

the rate of change function in f when $x_1 = \lim\limits_{h \to 0} \dfrac{f(x_1 + h) - f(x_1)}{h} = \lim\limits_{h \to 0} A(h)$ in a condition the limit should be existed.

Ex 5 :

Find the rate of change function in f when $x = x_1$ for each of the following , then find this rate at the given values of x .

a $f(x) = 3x^2 + 2$ when $x = 2$

b $f(x) = \dfrac{2}{x - 1}$ when $x = 3$

Ex 6 :

Find the average rate of change function in f where $f(x) = \frac{3}{x - 2}$ when x varies from x_1 to $x_1 + h$, then deduce the rate of change in f when $x = 5$.

Lesson 2 : Differentiation

The slope of the tangent at C = $\tan\theta = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$, if found

Ex 1 :

Find the slope of the tangent to the curve of the function f where $f(x) = 3x^2 - 5$ at point A (2, 7), then find the measure of the positive angle which the tangent makes with the positive direction of x-axis at point A to the nearest minute .

Ex 2 :

Find the slope of the tangent to the curve of the function f where $f(x) = x^3 - 4$ at point A (1, - 3), then find the measure of the positive angle which the tangent makes with the positive direction of x-axis at point A to the nearest minute.

The Derivative Function

For each value of the variable x in the domain of f is corresponded by a unique value to the rate of change in f , thus , the rate of change is a function in the variable x and called the "derivative function" or the first derivative of the function or first differential coefficient.

Definition

If $f :]\, a , b\, [\longrightarrow \mathbb{R}$ and $x \in]\, a , b[$, then **the derivative function f'** :

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ in a condition that this limit is existed.

Symbols of the derivative function :

If $y = f(x)$, then the first derivative of the function f is denoted by

y' or f' and read as " derivative of y " or derivative of f "

$\dfrac{dy}{dx}$ read as " dy by dx " or " derivative of y with respect to x"

Notice that the slope of the tangent to the curve of $y = f(x)$ at point $(x_1, f(x_1))$ is $f'(x_1)$

Ex 3 :

Find the derivative function of the function f where $f(x) = x^2 - x + 1$ using the definition of the derivative, then find the slope of the tangent at the point (- 2 , 7)

Ex 4 :

If $f(x) = 3x^2 + 4x + 7$, find the derivative of the function f using the definition of the derivative, then find the slope of the tangent at the point (- 1 , 6)

Differentiability of a function at a Point

It is said that the function f is differentiable when $x = a$ (where a belongs to the domain of the function) if and only if $f'(a)$ is existed where $f'(a) = \lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h}$

If a derivative is found for the function f at each point belongs to the interval $]\, c , d\, [$, we say that the function is differentiable in this interval.

In example (2) : we find that for each $x \in \mathbb{R}$ there is a derivative to the function f where $f'(x) = 2x - 1$ so the polynomial function is differentiable on $\mathbb{R}$

Ex 5 :

Prove that $f(x) = \frac{x-1}{x+1}$ is differentiable when $x = 2$

Ex 6 :

Prove that $f(x) = x^2 - x + 1$ is differentiable when $x = 1$

Right and Left Derivatives

If the function f is defined when x = a (where a belongs to the domain of the function), and the function rule on the right of a differs from its rule on the left of a , we discuss the differentiability when x = a by finding the right derivative of the function which is denoted by the symbol $f'(a^+)$ and the left derivative denoted by $f'(a^-)$ where :

The right derivative $f'(a^+) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$, **left derivative** $f'(a^-) = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$

The function is differentiable at a if and only if $f'(a^+) = f'(a^-)$, and the derivative of the function is denoted by the symbol $f'(a)$

Ex 7 :

Show that the function f where $f(x) = \begin{cases} x^2 & \text{when } x \leqslant 2 \\ x+2 & \text{when } x > 2 \end{cases}$ is not- differentiable when $x = 2$

Differentiation and Continuity

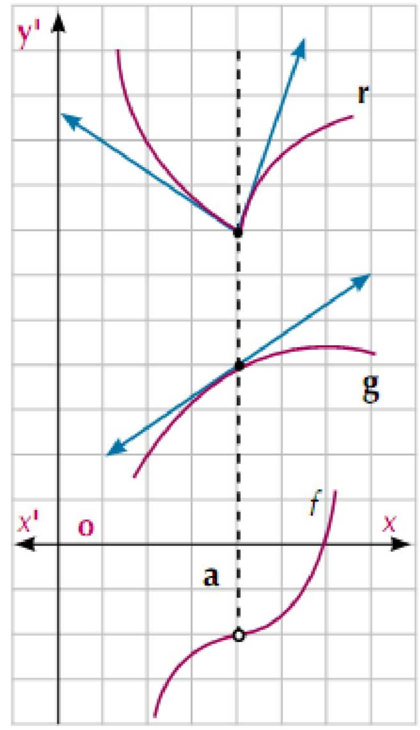
Definition

➢ If the function f where $y = f(x)$ is differentiable when $x = a$, then it is continuous at this point.

The opposite figure shows that:

1 - The continuity of a function at a point does not necessarily mean it is differentiable at the same point as in the two functions r and g.

2 - If the function is discontinuous when $x = a$, then the function is not-differentiable when $x = a$ as in the function f .



Ex 8 :

Discuss the differentiability of the function f at $x = 3$ where $f(x) = \begin{cases} 2x - 1 & \text{when } x < 3 \\ 7 - x & \text{when } x \geqslant 3 \end{cases}$

Ex 9 :

If the function f where $f(x) = \begin{cases} a\,x^2 + 1 & \text{when } x \leqslant 2 \\ 4\,x - 3 & \text{when } x > 2 \end{cases}$ is continuous at $x = 2$, find the value of the constant a , then discuss the differentiability of the function when $x = 2$

Ex 10 :

If $f(x) = a\,x^2 + b$ where a and b are two constants, find :

a) The first derivative of the function f at any point (x, y).

b) The two values of a and b if the slope of the tangent to the curve of the function at point (2, -3) lying on it equals 12.

Lesson 3 : Rules of differentiation

Derivative of a Function

1 - Derivative of the constant function

If $y = c$ **where:** $c \in \mathbb{R}$ **then:** $\frac{dy}{dx} = 0$

Notice :

$y = f(x) = c, \quad f(x + h) = c$

$\because \frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\therefore \frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{c - c}{h} = \text{zero} \ (h \neq 0)$

2 - Derivative of the funciton $f(x) = x^n$

If $y = x^n$ **where:** $n \in \mathbb{R}$ **then:** $\frac{dy}{dx} = n x^{n-1}$

If $y = x$ **then:** $\frac{dy}{dx} = 1$

If $y = a x^n$ **where:** $a, n \in \mathbb{R}$ **then:** $\frac{dy}{dx} = a n x^{n-1}$

Ex 1 :

Find $\frac{dy}{dx}$ in each of the following:

a) $y = -3$ b) $y = x^4$ c) $y = 5x$

d) $y = \frac{3}{x^2}$ e) $y = \sqrt{x^3}$

Ex 2 :

Find $\frac{dy}{dx}$ in each of the following:

a $y = -\sqrt{2}$ **b** $y = \frac{4}{3}\pi\ x^3$ **c** $y = \frac{-4}{x^5}$ **d** $y = \sqrt[3]{x^5}$

Derivative of the sum or difference between two functions

If z and g are two differentiable functions with respect to the variable x , then $z \pm g$ is also differentiable with respect to x and $\frac{d}{dx}(z \pm g) = \frac{dz}{dx} \pm \frac{dg}{dx}$, and in general :

If f_1 , f_2 ,...., f_n **are differentiable functions with respect to the variable** x **then:**

$$\frac{d}{dx}(f_1 \pm f_2 \pm f_3 \pm \dots \pm f_n)(x) = f_1'(x) \pm f'_2(x) \pm f'_3(x) \pm \dots \pm f'_n(x)$$

Ex 3 :

Find $\frac{dy}{dx}$ in each of the following:

a $y = 2x^6 + x^{-9}$

b $y = \frac{\sqrt{x} - 2x}{\sqrt{x}}$

Ex 4 :

Find $\frac{dy}{dx}$ if:

a $y = 3x^8 - 2x^5 + 6x + 1$

b $y = \frac{5}{x} + x\sqrt{x} + \sqrt{3}\,x - 4$

The derivative of the product of two functions:

if z and g are two differentiable functions with respect to the variable x, then the function (z . g) is also differentiable with respect to the variable x and $\frac{d}{dx}(z \cdot g) = z \frac{dg}{dx} + g \cdot \frac{dz}{dx}$

Ex 5 :

Find $\frac{dy}{dx}$ if $y = (x^2 + 1)(x^3 + 3)$, then find $\frac{dy}{dx}$ when $x = -1$

Ex 6 :

Find $\frac{dy}{dx}$ if $y = (4x^2 - 1)(7x^3 + x)$, then find $\frac{dy}{dx}$ when $x = 1$

Derivative of the quotient of two functions:

If z and g are two differentiable functions with respect to the variable x and $g(x) \neq 0$, then the function $(\frac{z}{g})$ is also differentiable with respect to the variable x

and $\frac{d}{dx}(\frac{z}{g}) = \frac{g\frac{dz}{dx} - z\frac{dg}{dx}}{g^2}$

i.e $(\frac{z}{g})' = \frac{g z' - z g'}{g^2}$

Ex 7 :

Find $\frac{dy}{dx}$ If $y = \frac{x^2 - 1}{x^3 + 1}$

Ex 8 :

Find $\frac{d\,y}{d\,x}$ if $y = \frac{x^3 + 2x^2 - 1}{x + 5}$

the composite function (Chain rule)

Theorem

If $y = f(z)$ is differentiable with respect to the variable z , and $z = r(x)$ is differentiable with respect to the variable x, then $y = f(r(x))$ is differentiable with respect to the variable x and: $\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

This theorem is known as the chain rule

Ex 9 :

If $y = (x^2 - 3x + 1)^5$, find $\frac{dy}{dx}$

Ex 10 :

If $y = \sqrt[3]{z}$, $z = x^2 - 3x + 2$, find $\frac{dy}{dx}$

Ex 11 :

If $y = 3z^2 - 1$, $z = \frac{5}{x}$, find $\frac{dy}{dx}$

Derivative of the function $[f(x)]^n$

If $z = [f(x)]^n$ where f is differentiable with respect to x and n is a real number ,

then: $\frac{dz}{dx} = n\,[f(x)]^{n-1} \times f'(x)$

Ex 12 :

Find $\frac{dy}{dx}$ if

a $y = (6x^3 + 3x + 1)^{10}$

b $y = (\frac{x - 1}{x + 1})^5$

Ex 13 :

Find $\frac{dy}{dx}$ if $y = (\frac{5x^2}{3x^2 + 2})^3$

Ex 14 :

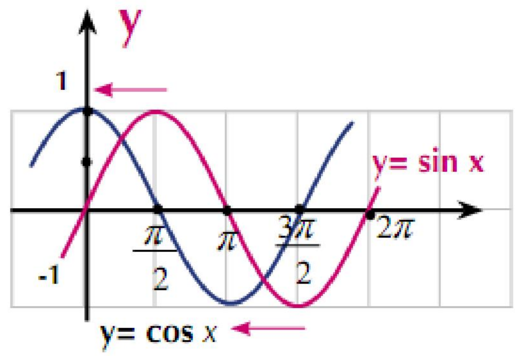
Find the values of x which make $f'(x) = 7$ in each of the following:

a $f(x) = x^3 - 5x + 2$

b $f(x) = (x - 5)^7$

Lesson 4 : Derivatives of trigonometric functions

The derivative of sine function



If $f(x) = \sin x$ then $f'(x) = \cos x$

$$\boxed{\frac{d}{dx}(\sin x) = \cos x}$$

In general If z is a differentiable function with respect to the variable x, then :

$$\frac{d}{dx}[\sin z] = \cos z \cdot \frac{dz}{dx} \qquad [\textbf{chain rule}]$$

Ex 1 :

Find $\frac{dy}{dx}$ for each of the following :

a) $y = 5 \sin x$ b) $y = x^3 \sin x$ c) $y = 2 \sin (3x + 4)$

1- Derivative of the cosine function

If $y = \cos x$ then $\frac{dy}{dx} = -\sin x$

2- Derivative of the tangent function

If $y = \tan x$ then $\frac{dy}{dx} = \sec^2 x$

Ex 2 :

Find the first derivative for each of the following :

a) $y = 2\cos x - \tan 5x$

b) $y = \tan(1 - x^2)$

c) $y = \cos^2(4x^2 - 7)$

Ex 3 :

find $\frac{dy}{dx}$ for each of the following :

a) $y = 2\tan 3x$

b) $y = 2\cos(4 - 3x^2)$

c) $y = 2\sin x \cos x$

d) $y = 2x\tan x$

e) $y = \tan^2 3x$

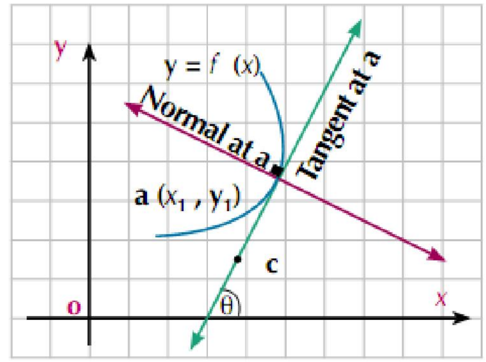
f) $y = \tan 4x^3$

Remember

$\sin^2 x + \cos^2 x = 1$

Lesson 5 : Applications on the derivative

Slope of the Tangent and the Normal to a Curve



- If m_1 and m_2 are the two slopes of two known straight lines ℓ_1 and ℓ_2, then:
- $\ell_1 \parallel \ell_2$ if and only if $m_1 = m_2$ (parallel condition)
- $\ell_1 \perp \ell_2$ if and only if $m_1 m_2 = -1$ (perpendicular condition)

slope of the normal on the curve of $y = f(x)$ at point (x_1, y_1) lying on it $= -\dfrac{1}{\left[\dfrac{dy}{dx}\right]_{(x_1, y_1)}}$

Ex 1 :

Find the points which lie on the curve of $y = x^3 - 4x + 3$ at which the tangent makes a positive angle of measure 135° with the positive direction of x axis .

Ex 2 :

Find the points which lie on the curve of $y = x^2 - 2x + 3$ at which the tangent to the curve is :

a Parallel to x-axis

b Perpendicular to the straight line $x - 4y + 1 = 0$

The equations of the Tangent and Normal to a Curve

If (x_1, y_1) is a point lying on the curve of the function f where $y = f(x)$,and m is the slope of the tangent at this point, then:

1 - The equation of the tangent to the curve at point (x_1, y_1) is :

$$y - y_1 = m(x - x_1)$$

2 - The equation of the normal to the curve at point (x_1, y_1) is :

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

Ex 3 :

Find the two equations of the tangent and normal to the curve of $y = 2x^3 - 4x^2 + 3$ at the point lying on the curve and whose abscissa = 2

Ex 4 :

Find the equation of the tangent to the curve of $y = 4x - \tan x$ at point $(\frac{\pi}{4}, f(\frac{\pi}{4}))$

Ex 5 :

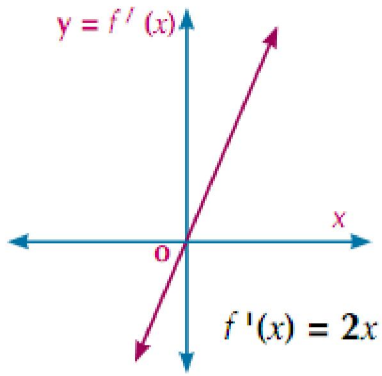
If the curve $y = ax^3 + b\,x^2$ touches the straight line $y = 8x + 5$ at point $(-1, -3)$, find the two values of a and b.

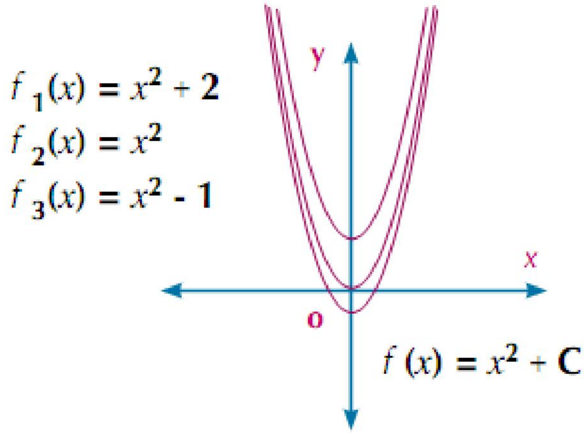
Ex 6 :

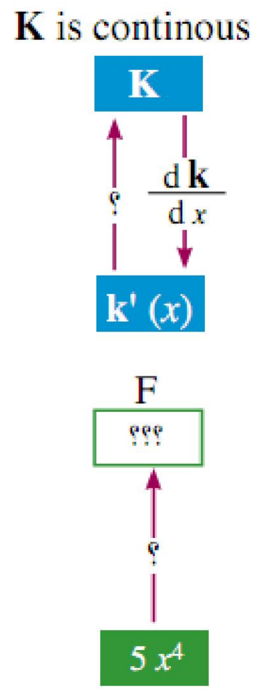
Find the value of the two constants a and b if the slope of the tangent to the curve of $y = x^2 + a\,x + b$ at point $(1, 3)$ lying on it equals 5

Lesson 6 : Integration

Antiderivative







Definition

It is said that the function F is antiderivative to the function f, if $F'(x) = f(x)$ for each x in the domain of f.

Ex 1:

Prove that the function f where $F(x) = \frac{1}{2}x^4$ is an antiderivative to the function f where $f(x) = 2x^3$.

Ex 2 :

Show that the function F where $F(x) = \frac{1}{2}x^6$ is an antiderivative to the function f where $f(x) = 3x^5$

Indefinite Integral

The set of the antiderivatives to the function f is called an indefinite integral of this function and is denoted by the symbol $\int f(x)\, dx$ [and read: integral of $f(x)$ with respect to x]

Definition

If $F^{\backprime}(x) = f(x)$, then $\int f(x) \, f x = F(x) + C$

where C is an arbitrary constant.

Notice : $\frac{d}{dx}(x^3+5) = 3x^2$ $\therefore \int 3x^2\, dx = x^3 + C$

$\frac{d}{dx}(x^5-3) = 5x^4$ $\therefore \int 5x^4\, dx = x^5 + C$

$\frac{d}{dx}(2x^7) = 14x^6$ $\therefore \int 14x^6\, dx = 2x^7 + C$

Ex 3 :

Check the correctness for each of the following:

a) $\int x^7\, dx = \frac{1}{8}x^8 + C$

b) $\int \frac{x}{\sqrt{1+x^2}} dx = \sqrt{1+x^2} + C$

Rule:

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ where C is a constant , n is a rational number and $n \neq -1$

Ex 4 : Find :

a $\int x^5 \, dx$

b $\int x^{-3} \, dx$

c $\int x^{\frac{2}{5}} \, d x$

d $\int \frac{1}{\sqrt[5]{x^3}} \, d x$

Ex 5 :

Find:

a $\int x^8 \, d x$

b $\int x^{\frac{2}{3}} \, dx$

c $\int \sqrt[7]{x^5} \, d x$

d $\int 7x^{\frac{7}{9}} \, d x$

Properties of Integration

If f and g are integrable functions on an interval , then:

1- $\int a f(x)\, dx = a \int f(x)\, dx$ where a is a constant $\neq 0$

2- $\int [f(x) \pm r(x)]\, dx = \int f(x)\, dx \pm \int r(x)\, dx$

Ex 6 :

Find: **a** $\int (4x + 3x^2)\, dx$ **b** $\int \frac{(x^2+2)^2}{x^2}\, dx$

Ex 7 :

Find:

a $\int (2 + \sqrt{x} + \frac{1}{\sqrt{x}})\, dx$ **b** $\int (\frac{1}{x^2} + \sqrt{x} + 3)\, dx$

Some rules of integration

1- $\int (a\,x + b)^n \; dx = \frac{1}{a} \times \frac{(a\,x + b)^{n+1}}{n+1} + C \; , \; n \neq -1$

2- $\int \; [f(x)]^n \; f'(x) \; dx = \frac{[f(x)]^{n+1}}{n+1} + C$, where C is a constant and n is a rational number $\neq -1$

Ex 8 :

Find:

a $\int \left((3 - 2x)^5 + 3) \right) \; d\,x$

b $\int \frac{x + 3}{(x - 2)^4} \; d\,x$

c $\int (x^2 - 3x + 5)^{-7} (2\,x - 3) \, d\,x$

d $\int (3x^2 - 2x + 1)^{11} (3x - 1) \, d\,x$

Integration of Some Trignometric Functions

1- $\int \sin x \, dx = -\cos x + C$

2- $\int \cos x \, dx = \sin x + C$

3- $\int \sec^2 x \, dx = \tan x + C$

where C is an arbitrary constant

Remember

Some trigonometric relations

(a) $\cos^2 x + \sin^2 x = 1$

(b) $\cos^2 x - \sin^2 x = \cos 2x$

(c) $1 + \tan^2 x = \sec^2 x$

(d) $2 \sin x \cos x = \sin 2x$

Ex 9 :

Find the following integrations:

(a) $\int (x - \sin x) \, dx$

(b) $\int \left(4 \cos x + \frac{1}{\cos^2 x} + 1\right) dx$

Important Corollaries :

1 - $\int \sin(ax + b) \, dx = -\frac{1}{a} \cos(ax + b) + C$

2 - $\int \cos(ax + b) \, dx = \frac{1}{a} \sin(ax + b) + C$

3 - $\int \sec^2(ax + b) \, dx = \frac{1}{a} \tan(ax + b) + C$ where C is an arbitrary constant

Ex 10 :

Find:

a $\int \cos(2x + 3)\ dx$

b $\int (\sec^2 \frac{x}{2} - \sin(\frac{\pi}{4} - x))\ dx$

Ex 11 :

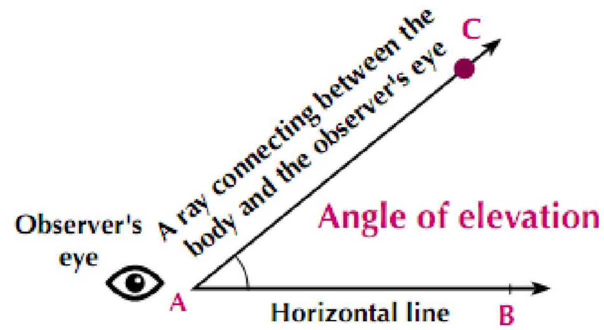
Find :

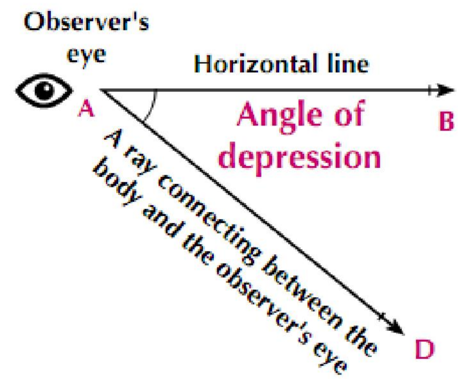
a $\int \sin(3x - 5) f\ dx$

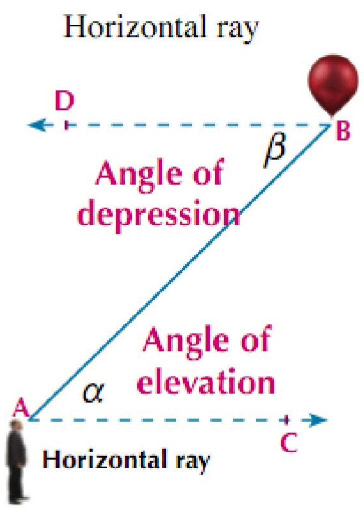
b $\int \cos(\frac{x}{3} - 2)\ dx$

Unit 4 : Trigonometry

Lesson 1 : Angles of elevation and depression

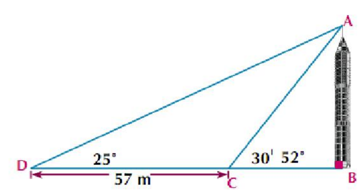






Ex 1 :

From a point on the ground surface, a man observed the top of a tower of an angle of elevation of 25°, then he walked straight a head for 57 m at the horizontal level toward the tower base to find that the angle of elevation of the tower top is **30°52`** Find the height of the tower to the nearest meter.



Ex 2 :

From a point on the ground surface a man observed the top of a tower at an angle of elevation of 20°, He walked on a horizontal way in the direction of the tower base for 50 meters, the measurement of the angle of elevation of the tower top is 42°. Find the height of the tower to the nearest meter .

Ex 3 :

From the top a rock of height 80 meters, the two angles of depression of the top and the base of a tower were measured to give 24° and 35° respectively. Find the height of the tower to the nearest meters known that the two bases of the rock and tower are in the same horizontal level.

Ex 4 :

From point A on a riverbank, a man observed the position of a home at point B on the other riverbank to find it in the direction of $20°$ North of the east. As he walks parallel to the riverbank in the direction of East for a distance of 300 meters to reach point C, he found point B in the direction of $46°$ North of the east. Find the width of the river to the nearest meter known that the two riverbanks are parallel and points A , B and C are at the same horizontal level.

Ex 5 :

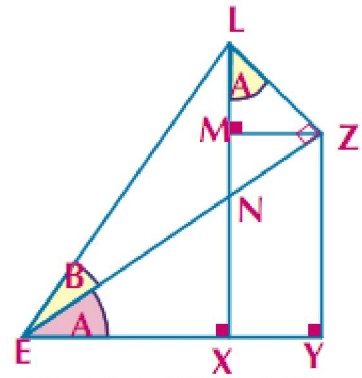
A man measured the angle of elevation of a hill top from a point on the ground surface to find it $22°$. As he ascends the hill for 500 meters on a road inclined to the horizontal by an angle of measurement $7°$, he found the measure of the angle of elevation of the hill top is $64°$. Find the height of the hill to the nearest meter .

Lesson 2 : Trigonometric functions of sum and difference of the measures of two angles

Trigonometric functions of sum and difference of the measures of two angles

From the opposite figure : (proof is not required)

Notice that m(∠A) = m (∠Z L M). Why?



(notice that: Z Y = M X - why?)

$$\sin(A+B) = \frac{LX}{LE} = \frac{MX}{LE} + \frac{LM}{LE}$$

$$= \frac{ZY}{LE} + \frac{LM}{LE}$$

$$= \frac{ZY}{LE} \times \frac{ZE}{ZE} + \frac{LM}{LE} \times \frac{LZ}{LZ}$$

$$= \frac{ZY}{ZE} \times \frac{ZE}{LE} + \frac{LM}{LZ} \times \frac{LZ}{LE}$$

$$= \sin A \times \cos B + \cos A \times \sin B$$

Then $\boxed{\sin(A+B) = \sin A \cos B + \cos A \sin B}$

By putting (- B) instead of B, we get:

$\sin[A + (-B)] = \sin A \cos(-B) + \cos A \sin(-B)$

$\boxed{\sin(A-B) = \sin A \cos B - \cos A \sin B}$

Use the same figures to prove:

$\boxed{\cos(A+B) = \cos A \cos B - \sin A \sin B}$

then , we deduce that: $\boxed{\cos(A-B) = \cos A \cos B + \sin A \sin B}$

Remember

$\sin(-A) = -\sin A$

$\cos(-A) = \cos A$

$\tan(-A) = -\tan A$

Ex 1 :

Find:

a) $\sin 75°$ b) $\cos 15°$ what do you notice?

Ex 2 :

Find.

a $\cos 105^\circ$ **b** $\sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ$

c $\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ$

Remember

$\sin(180-A) = \sin A$

$\cos(180-A) = -\cos A$

$\sin(180+A) = -\sin A$

$\cos(180+A) = -\cos A$

Ex 3 :

If $\sin A = \frac{3}{5}$ where $90^\circ < A < 180^\circ$, $\cos B = \frac{-5}{13}$

where $180^\circ < B < 270^\circ$

find $\cos(A - B)$, $\sin(A + B)$

Ex 4 :

In the triangle A B C, $\cos A = \frac{-3}{5}$ and $\sin B = \frac{5}{13}$, Find sin C without using the calculator.

Tangent function of sum and difference of the measures of two angles

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

By dividing the nominator and denominator by $\cos A \cos B \neq 0$, then:

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

when putting (-B) instead of B, then:

$$\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}}$$

where A and B $\neq \frac{\pi}{2}(2n+1)$ and $n \in Z$

Ex 5 :

Without using the calculator , prove that:

a $\tan 50^\circ = \frac{1 + \tan 5^\circ}{1 - \tan 5^\circ}$

b $\tan(45^\circ - A) = \frac{\cos A - \sin A}{\cos A + \sin A}$

Remember

$\tan A = \frac{\sin A}{\cos A}$

Ex 6 :

If A, B and C are the measures of the angles of a triangle where $\tan B = \frac{4}{3}$, $\tan C = 7$, prove that $A = 45^\circ$

Ex 7 :

Find the solution set for each of the following equations where $0^\circ < x < 360^\circ$

a $\tan x + \tan 20^\circ + \tan x \tan 20^\circ = 1$ **b** $\sin(x + 30^\circ) = 2\cos x$

Lesson 3 : The trigonometric functions of the double-angle

The trigonornetric functions of the double - angle

You know that:

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

(by putting B = A)

$\therefore \sin(A+A) = \sin A \cos A + \cos A \sin A$

$\therefore \sin 2A = 2\sin A \cos A$ for each $A \in R$ (1)

Similarly:

$\cos 2A = \cos^2 A - \sin^2 A$

for each $A \in R$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ where $\tan A$ is defined, $\tan^2 A \neq 1$

Remember

The basic relations among the trigonometric functions:

$\sin^2 C + \cos^2 C = 1$

$\tan^2 C + 1 = \sec^2 C$

$\cot^2 C + 1 = \csc^2 C$

$\csc C = \dfrac{1}{\sin C}$

$\sec C = \dfrac{1}{\cos C}$

$\tan C = \dfrac{\sin C}{\cos C}$

$\cot C = \dfrac{1}{\tan C}$

$\tan C = \dfrac{1}{\cot C}$

Ex 1 :

If you know $\sin A = \frac{4}{5}$ where $0° < A < 90°$, find the value for each of the following without using the calculator:

(a) $\sin 2A$ (b) $\cos 2A$ (c) $\tan 2A$

Ex 2 :

If $\cos A = \frac{4}{5}$, $0° < A < 90°$, find the values for each of the following without using the calculator:

a $\sin 2A$ **b** $\cos 2A$ **c** $\tan 2A$

Ex 3 :

Find the value for each of the following, without using the calculator , :

a $2 \sin 15° \cos 15$ **b** $2\cos^2 22° 30' - 1$

The trigonornetric functions of the half-angle

$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$, $\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$, $\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$ **where** $\cos A \neq -1$

the sign is determined according to the quadrant at which the angle $\frac{A}{2}$ lies in

Ex 4 :

Find the value for each of the following Without using the calculator :

a $\sin \frac{\theta}{2}$ known that , $\sin \theta = -\frac{4}{5}$, $180° < \theta < 270°$

b $\cos 75°$

c $\tan 22° 30'$

Ex 5 :

Prove the correctness of the identity: $\csc 2x + \cot 2x = \cot x$, then use the previous identity to find the value of $\cot 15°$.

Ex 6 :

If $4 \cos 2C + 3 \sin 2C = 0$, find without using the calculator the value of $\tan C$, where C is the measurement of a positive acute angle.

Ex 7 :

Find the values of x included between 0 and 2π which satisfy the following equations:

a $\sin 2x = \sin x$ **b** $\cos^2 x - \sin^2 x = -\frac{1}{2}$ **c** $\tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} = 1$

Lesson 4 : Heron's formula

Finding the surface area of the triangle in terms of its side lengths

I.e: the surface area of the triangle whose side lengths are a, b and c is:

$$\Delta = \sqrt{p(p-a)(p-b)(p-c)}$$ where P is half of the triangle perimeter

Ex 1 :

Find the surface area of the triangle whose side lengths are 6, 8 and 10 centimetres using Heron's formula

Ex 2 :

Find the surface area of the triangle A B C in which:
a= 5cm , b = 12 cm , c = 13cm using Heron's formula.

Ex 3 :

Find the surface area of the triangle A B C in each of the following cases:

a) $a = 15cm$, $b = 12cm$, $c = 9$ cm

b) $b = 16cm$, $c = 20$ cm, $m(\angle A) = 60^\circ$

c) $a = 16cm$, $b= 18cm$, $c = 24$ cm

d) $a= 32cm$, $b = 36$, $c = 30$ cm